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Symmetries of the Euler compressible flow equations for general equation of state

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Abstract

The Euler compressible flow equations exhibit different Lie symmetries depending on the equation of state (EOS) of the medium in which the flow occurs. This means that, in general, different types of similarity solution will be available in different flow media. We present a comprehensive classification of all EOS's to which the Euler equations apply, based on the Lie symmetries admitted by the corresponding flow equations, restricting to the case of 1-D planar, cylindrical, or spherical geometry. The results are conveniently summarized in tables. This analysis also clarifies past work by Axford and Ovsiannikov on symmetry classification.

1 Introduction

The objective of this study is to express the symmetries of the 1-D Euler compressible flow equations, for *any* equation of state to which the Euler compressible flow equations apply. The classification builds on a well-known analysis of the Euler equations by Ovsiannikov. He classified the symmetries relevant to a full 3-D geometry, and our goal is to adapt this analysis to the case of 1-D planar, cylindrical, or spherical geometry, meaning that the flow velocity, density, and specific energy should be a function of a single space coordinate and time. The end result of this analysis is Tables ??, ??, and ??, which give the complete set of symmetries for various materials.

Axford did an analysis similar to this one as part of his work on the Noh problem. His focus was different from ours, but his results overlap with and even appear to contradict ours. At the end of the paper, we present a detailed analysis of Axford's approach as it relates to our own, showing that the contradiction is only apparent.

2 Hydrodynamic equations in terms of the adiabatic bulk modulus

The governing equations for an inviscid, non-conducting, compressible fluid in n spacial dimensions are given by the system of PDE's

$$\rho d_t u + \nabla p = 0 \tag{1}$$

$$d_t \rho + \text{div } u = 0 \tag{2}$$

$$d_t S(p, \rho) = 0 \tag{3}$$

where d_t is the total derivative, and ρ , u , and p are the density, velocity, and pressure respectively. The function S is the entropy, which we assume satisfies $\partial_p S|_\rho \neq 0$, in which case we define

$$A = -\rho \frac{\partial_\rho S|_p}{\partial_p S|_\rho},$$

and rewrite the third equation as

$$d_t p + A \text{div } u = 0.$$

We call A the *adiabatic bulk modulus*. Note that knowing the bulk modulus is almost equivalent to knowing the equation of state. If we let ϵ be the specific internal energy, then the equation of state can be recovered from the bulk modulus via the relation

$$A(p, \rho) \frac{\partial \epsilon}{\partial p} + \rho \frac{\partial \epsilon}{\partial \rho} = \frac{p}{\rho}$$

or alternatively one can recover $p(\rho, \epsilon)$ via

$$A(p, \rho) = \rho \frac{\partial p}{\partial \rho} + \frac{p}{\rho} \frac{\partial p}{\partial \epsilon},$$

although either of these approaches requires in general the introduction of an arbitrary function parameter to get the general solution. This paper uses the bulk modulus rather than the EOS because the calculations are greatly simplified. While some effort is made to translate the results into the language of EOS, this is sometimes impossible because solving the PDE that relates the two may not have simple closed-form solution. Fortunately, if the reader wishes to apply these results to a particular EOS, it is often a simple matter to plug the EOS into the conversion PDE, either numerically or symbolically, and recover the corresponding bulk modulus.

We assume throughout that $A \neq 0$, except where specified. We also assume throughout that all functions encountered are smooth.

Finally, converting equations ??-?? to 1-D planar, cylindrical, and spherical geometries yields

$$\rho_t + u\rho_r + \rho \left(u_r + \frac{(n-1)u}{r} \right) = 0 \quad (4)$$

$$\rho(u_t + uu_r) + p_r = 0 \quad (5)$$

$$p_t + up_r + A(p, \rho) \left(u_r + \frac{(n-1)u}{r} \right) = 0, \quad (6)$$

where n is set to 1 for planar, 2 for cylindrical, and 3 for spherical geometries, respectively.

3 The 3-D case

As stated above, we seek to determine, for any given A , what symmetries are possessed by the Euler equations. In the fully 3-D case, this problem was first solved by Ovsiannikov, who used a Lie group method to produce the following list of symmetries:

$$\phi_1 \cdot \partial = \partial_t \quad (7)$$

$$\phi_2 \cdot \partial = t\partial_t + x_i\partial_{x_i} \quad (8)$$

$$\phi_3^i \cdot \partial = \partial_{x_i} \quad (9)$$

$$\phi_4^i \cdot \partial = t\partial_{x_i} + \partial_{u^i} \quad (10)$$

$$\phi_5^{ik} \cdot \partial = x^k\partial_{x_i} - x^i\partial_{x_k} + u^k\partial_{u^i} - u^i\partial_{u^k} \quad (11)$$

$$\phi_6 \cdot \partial = t\partial_t - u^i\partial_{u^i} + 2\rho\partial_\rho \quad (12)$$

$$\phi_7 \cdot \partial = p\partial_p + \rho\partial_\rho \quad (13)$$

$$\phi_8 \cdot \partial = \partial_p \quad (14)$$

$$\phi_9^{p_0} \cdot \partial = t^2\partial_t + tx^i\partial_{x_i} + (x^i - tu^i)\partial_{u^i} - (n+2)t(p+p_0)\partial_p - nt\rho\partial_\rho. \quad (15)$$

where x_i is the i -th spacial variable, and u_i is the i -th component of the velocity. The summation convention on repeated indices is observed.

Of these symmetries, the first five are valid regardless of the choice of A , and the remaining four may or may not be present, depending on A . Exactly when each of the extra symmetry groups is manifested is summarized in Tables ?? and ??.

A	r	Operators
$(p+p_0)f((p+p_0)^{1+2m}\rho^{-1})$	1	$m\phi_6 + \phi_7 + p_0\phi_8$
$f(\rho e^{-2ap})$	1	$\phi_6 + \frac{1}{a}\phi_8$
$f(\rho)$	1	ϕ_8
$f(p)$	1	ϕ_6
$A_0\rho^m$	2	$\phi_8, (m-1)\phi_6 - 2m\phi_7$
$\gamma(p+p_0)$	2	$\phi_6, \phi_7 + p_0\phi_8$
$\frac{n+2}{n}(p+p_0)$	3	$\phi_6, \phi_7 + p_0\phi_8, \phi_9^{p_0}$

Table 1: Symmetry classification of bulk moduli (3-D, compact). Each class of bulk moduli on the left possesses the symmetry operators given on the right, in addition to ϕ_1 - ϕ_5 , which are admitted by all bulk moduli. Here, r is the number of extra symmetry generators, f is an arbitrary function, and m , p_0 , and a are arbitrary constants, $a \neq 0$.

A	1	2	3	4	5	6	7	8
$f(p, \rho)$	∂_t	$t\partial_t + x_i\partial_{x_i}$	∂_{x_i}	$t\partial_{x_i} + \partial_u$	$x^k\partial_{x_i} - x^i\partial_{x_k} + u^k\partial_{u^i} - u^i\partial_{u^k}$	$mt\partial_t - mu^i\partial_{u^i} + (2m+1)\rho\partial_\rho + (p+p_0)\partial_p$		
$(p+p_0)f((p+p_0)^{1+2m}\rho^{-1})$	∂_t	$t\partial_t + x_i\partial_{x_i}$	∂_{x_i}	$t\partial_{x_i} + \partial_u$	$x^k\partial_{x_i} - x^i\partial_{x_k} + u^k\partial_{u^i} - u^i\partial_{u^k}$	$t\partial_t - u^i\partial_{u^i} + 2\rho\partial_\rho + \frac{1}{a}\partial_p$		
$f(\rho e^{-2ap})$	∂_t	$t\partial_t + x_i\partial_{x_i}$	∂_{x_i}	$t\partial_{x_i} + \partial_u$	$x^k\partial_{x_i} - x^i\partial_{x_k} + u^k\partial_{u^i} - u^i\partial_{u^k}$	∂_p		
$f(\rho)$	∂_t	$t\partial_t + x_i\partial_{x_i}$	∂_{x_i}	$t\partial_{x_i} + \partial_u$	$x^k\partial_{x_i} - x^i\partial_{x_k} + u^k\partial_{u^i} - u^i\partial_{u^k}$	$t\partial_t - u^i\partial_{u^i} + 2\rho\partial_\rho$	∂_p	
$f(p)$	∂_t	$t\partial_t + x_i\partial_{x_i}$	∂_{x_i}	$t\partial_{x_i} + \partial_u$	$x^k\partial_{x_i} - x^i\partial_{x_k} + u^k\partial_{u^i} - u^i\partial_{u^k}$	$(m-1)t\partial_t - (m-1)u^i\partial_{u^i} - 2\rho\partial_\rho - 2mp\partial_p$	$(p+p_0)\partial_p + \rho\partial_\rho$	
$A_0\rho^m$	∂_t	$t\partial_t + x_i\partial_{x_i}$	∂_{x_i}	$t\partial_{x_i} + \partial_u$	$x^k\partial_{x_i} - x^i\partial_{x_k} + u^k\partial_{u^i} - u^i\partial_{u^k}$	$t\partial_t - u^i\partial_{u^i} + 2\rho\partial_\rho$	$(p+p_0)\partial_p + \rho\partial_\rho$	
$\gamma(p+p_0)$	∂_t	$t\partial_t + x_i\partial_{x_i}$	∂_{x_i}	$t\partial_{x_i} + \partial_u$	$x^k\partial_{x_i} - x^i\partial_{x_k} + u^k\partial_{u^i} - u^i\partial_{u^k}$	$t\partial_t - u^i\partial_{u^i} + 2\rho\partial_\rho$	$(p+p_0)\partial_p + \rho\partial_\rho$	
$\frac{n\pm 2}{n}(p+p_0)$	∂_t	$t\partial_t + x_i\partial_{x_i}$	∂_{x_i}	$t\partial_{x_i} + \partial_u$	$x^k\partial_{x_i} - x^i\partial_{x_k} + u^k\partial_{u^i} - u^i\partial_{u^k}$	$t\partial_t - u^i\partial_{u^i} + 2\rho\partial_\rho$	$(p+p_0)\partial_p + \rho\partial_\rho$	$\phi_9^{p_0}$

Table 2: Symmetry classification of bulk moduli (3-D, expanded). Each class of bulk moduli on the left has the symmetry generators given in its row. Here, f is an arbitrary function, n is the spacial dimension, and A_0, p_0, a and m are arbitrary constants, $A_0, a \neq 0$.

A	q	Operators
$(p + p_0)f((p + p_0)^{1+2m}\rho^{-1})$	1	$m\zeta_5 + \zeta_6 + p_0\zeta_7$
$f(\rho e^{-2ap})$	1	$\zeta_5 + \frac{1}{a}\zeta_7$
$f(\rho)$	1	ζ_7
$f(p)$	1	ζ_5
$A_0\rho^m$	2	$\zeta_7, (m-1)\zeta_5 - 2m\zeta_6$
$\gamma(p + p_0)$	2	$\zeta_5, \zeta_6 + p_0\zeta_7$
0	2	ζ_5, ζ_9
$\frac{n+2}{n}(p + p_0)$	3	$\zeta_5, \zeta_6 + p_0\zeta_7, \zeta_8^{p_0}$

Table 3: Symmetry classification of bulk moduli (compact form). Each class of bulk moduli on the left is generated by the symmetry operators given on the right, in addition to the symmetries possessed by all bulk moduli. Here, q is the number of extra symmetry generators, n is 1,2, or 3 for planar, cylindrical, or spherical symmetry, respectively, f is an arbitrary nonzero function, a is a nonzero constant, p_0 is an arbitrary constant, and m is a constant which must be nonzero in the case $A = A_0\rho^m$. The definition of the ζ_i is given in equations ??-??.

The physical meaning of these symmetries is worth noting. $\phi_1, \phi_3^i, \phi_4^i$, and ϕ_5^{ik} are standard coordinate shifts. It is tempting to say that ϕ_2, ϕ_6 , and ϕ_7 are just scaling spacetime, time, and mass, respectively, but this is not true. ϕ_2 fails to scale pressure as expected, and the correct space, time, and mass scalings are $\phi_2 - \phi_6 - \phi_7, \phi_6 - 2\phi_7$, and ϕ_7 , respectively. Since ϕ_6 and ϕ_7 do not apply in the case of general EOS, we see that the interpretation of these generators as unit scalings is not in general valid and that this fact arises from the arbitrary nature of the bulk modulus, which may, for instance, contain dimensional constants that would have to be adjusted by the same scaling as the variables. Finally, ϕ_8 reflects independence of the speed of sound from pressure. ϕ_9 has no obvious physical meaning, but related symmetries appear in other hyperbolic equations and sometimes have a clearer meaning in terms of in the Lorenz metric. We refer the reader to a treatment of a related symmetry in Olver, page 125.

4 Collapse to 1-D symmetry

The symmetries in the previous section apply to full 3-D geometry. When we wish to work in 1-D geometry, not all of these symmetries will apply, since applying some symmetry operators tends to bread the 1-D symmetry, especially in the cylindrical and spherical cases, as we shall see. Conversely, some symmetries, such as rotation in spherical symmetry, have no effect at all. The symmetries which will have useful, nontrivial effects in 1-D geometries are

$$\zeta_1 \cdot \partial = \partial_t \tag{16}$$

$$\zeta_2 \cdot \partial = t\partial_t + r\partial_r \tag{17}$$

$$\zeta_3 \cdot \partial = \partial_r \tag{18}$$

$$\zeta_4 \cdot \partial = t\partial_r + \partial_u \tag{19}$$

$$\zeta_5 \cdot \partial = t\partial_t - u\partial_u + 2\rho\partial_\rho \tag{20}$$

$$\zeta_6 \cdot \partial = p\partial_p + \rho\partial_\rho \tag{21}$$

$$\zeta_7 \cdot \partial = \partial_p \tag{22}$$

$$\zeta_8^{p_0} \cdot \partial = t^2\partial_t + tr\partial_r + (r - tu)\partial_u - (n+2)t(p+p_0)\partial_p - nt\rho\partial_\rho \tag{23}$$

$$\zeta_9 \cdot \partial = f(p)\partial_p + \rho f'(p)\partial_\rho. \tag{24}$$

In analogy with the 3-D case, ζ_1 - ζ_4 apply regardless of the choice of A , and the rest may arise or not, depending on the choice of A . In addition, ζ_3 and ζ_4 are only valid in the case of planar symmetry. The exact situation in which each of zeta 5-9 apply is shown in Tables ?? and ??. Also, we have included Table ?? to help convert between EOS and A , although the correspondence given is not complete since some of the cases proved too general to yield a closed-form solution to the PDE relating EOS with A . Notice that it is permitted to add an arbitrary function of (generalized) entropy to the EOS. Observe that ζ_8 and ζ_9 occur only in very special cases which may actually occur physically. For instance, the ideal gas gamma for a noble gas at 20 degrees celsius approximately matches the EOS for ζ_8 in the spherical case. Finally, we have included the case where $A = 0$ in these tables for the sake of completeness (This is why ζ_9 appears here but was not in the 3-D table).

There is one unusual symmetry that applies only in one ideal gas case, namely

$$\zeta_8 \cdot \partial = t^2\partial_t + tr\partial_r + (r - tu)\partial_u - (n+2)tp\partial_p - nt\rho\partial_\rho,$$

where we have set $p_0 = 0$. Integrating this gives the functions t, r, u, p, ρ , in terms of a parameter s . They are

$$t(s) = \frac{t_0}{1 - st_0} \quad (25)$$

$$r(s) = \frac{r_0}{1 - st_0} \quad (26)$$

$$u(s) = u_0 + s(r_0 - t_0 u_0) \quad (27)$$

$$p(s) = p_0(1 - st_0)^{n+2} \quad (28)$$

$$\rho(s) = \rho_0(1 - st_0)^n. \quad (29)$$

$$(30)$$

The physical meaning of this symmetry is not obvious. It should also be noted that the symmetry can only apply to situations where time is bounded, since a singularity occurs when $s = \frac{1}{t_0}$.

5 Applicability to various test problems

Symmetry analysis is useful for working out similarity solutions in analytic or reduced form, which can then be used as test problems for hydrocodes. Each of the symmetries defined above may be useful in these endeavors, with the warning that some of them may alter the initial/boundary conditions in unacceptable ways. For instance, any of the transformations that scales density would be inappropriate for the Noh, Sedov, and Guderley problems, since the definitions of these problems requires a constant, nonzero density to persist in regions ahead of the shocks. Sometimes this can be fixed by taking linear combinations of the generators so that the density cancels (indeed, Axford uses this trick in his paper). Different test problems will yield different applicable symmetries.

6 Comparison with Axford

Axford does an analysis similar to Ovsiannikov's and the one in this paper, but focusing specifically on applications to the Noh test problem. Since the Noh problem was already known to have a similarity solution when the material is an ideal gas, Axford claims to find the set of all bulk moduli that admit the same symmetry generators as the ideal gas EOS and then shows that any such bulk modulus admits a similarity solution of the Noh problem. But there is a minor error in his explanation that makes it appear as though his analysis contradicts this one. The ideal gas bulk modulus admits the following symmetry group.

$$\zeta_1 \cdot \partial = \partial_t \quad (31)$$

$$\zeta_2 \cdot \partial = t\partial_t + r\partial_r \quad (32)$$

$$\zeta_3 \cdot \partial = \partial_r \quad (33)$$

$$\zeta_4 \cdot \partial = t\partial_r + \partial_u \quad (34)$$

$$\zeta_5 \cdot \partial = t\partial_t - u\partial_u + 2\rho\partial_\rho \quad (35)$$

$$\zeta_6 \cdot \partial = p\partial_p + \rho\partial_\rho \quad (36)$$

But, to preserve the boundary conditions of the Noh problem, it is necessary to consider only those symmetries for which the coefficient of ζ_5 is $-\frac{1}{2}$ the coefficient of ζ_6 . He also tosses out symmetries involving ζ_3 and ζ_4 to ensure that symmetry is preserved in the curvilinear case. Then he tosses out ζ_1 , leaving only the two symmetries ζ_2 and $\zeta_5 - 2\zeta_6$ for use in constructing a similarity solution. Finally, Axford focuses exclusively on the generator ζ_2 for the rest of his solution. Thus, in the end, Axford relies on only one symmetry to solve his problem. So Axford's presentation of the classical ideal gas solution involves multiple reductions to smaller and smaller groups.

Axford generalizes this procedure by finding all bulk moduli that admit the two-parameter group generated by ζ_2 and $\zeta_5 - 2\zeta_6$. These bulk moduli are of the form $pf(\rho)$ or $(p + p_0)f(\rho)$, up to an equivalence transformation, and they include the ideal gas, stiff gas, and certain Gruneisen EOSs. Our analysis agrees with this result, since this class of bulk moduli is included in the first line of ?? and ??.

There is, however, an apparent contradiction, due in part to a minor mistake in the way Axford explains himself. While he found the bulk moduli that admit ζ_2 and $\zeta_5 - 2\zeta_6$, what he says is that he found the bulk moduli that admit the generators ??-??. While this mistake does not materially change his analysis, it does introduce an apparent contradiction, since it appears that he claims that bulk moduli of the form $pf(\rho)$ possess 4 symmetries (6 in the planar case), while in fact they possess only 3 (5 in the planar case). Thus, Ovsiannikov's, Axford's, and this papers' classification theorems do indeed agree.

7 Number of symmetry generators

The fact that only one symmetry generator is needed to reduce to ODEs is sometimes not appreciated, partly because the search for the correct generator often involves selecting carefully from a large group. Indeed, Axford's approach was to only work with materials that admitted all the symmetries of an ideal gas, which, as we have shown, possesses quite a lot of symmetry compared to other materials. It is nevertheless true in principle that one symmetry generator can be enough to reduce to ODEs, since we only want to reduce the number of variables by one. What is not so clear is how to choose the symmetry generator so that it yields good similarity variables for a given problem.

8 Conclusion

A complete classification of the different symmetry groups that the Euler equations may possess was given in the case of 1-D planar, cylindrical, or spherical symmetry. Furthermore, we reconcile these results with those given by Axford.

9 References

- L.V. Ovsiannikov, *Group Analysis of Differential Equations*, New York: Academic Press, 1982.
- Roy A. Axford, "Solutions of the Noh problem for various equations of state using Lie groups," Los Alamos National Laboratory, LA 13497, 1998.
- Peter J. Olver, *Applications of Lie Groups to Differential Equations*, New York: Springer Verlag, 1993.

A	1	2	3*	4*	5	6	7
$f(p, \rho)$	∂_t	$t\partial_t + r\partial_r$	∂_r	$t\partial_r + \partial_u$			
$(p + p_0)f((p + p_0)^{1+2m}\rho^{-1})$	∂_t	$t\partial_t + r\partial_r$	∂_r	$t\partial_r + \partial_u$	$mr\partial_r + mu\partial_u - (2m + 1)\rho\partial_\rho - (p + p_0)\partial_p$		
$f(\rho e^{-2ap})$	∂_t	$t\partial_t + r\partial_r$	∂_r	$t\partial_r + \partial_u$	$r\partial_r + u\partial_u - 2\rho\partial_\rho - \frac{1}{a}\partial_p$		
$f(\rho)$	∂_t	$t\partial_t + r\partial_r$	∂_r	$t\partial_r + \partial_u$	∂_p		
$f(p)$	∂_t	$t\partial_t + r\partial_r$	∂_r	$t\partial_r + \partial_u$	$r\partial_r + u\partial_u - 2\rho\partial_\rho$		
$A_0\rho^m$	∂_t	$t\partial_t + r\partial_r$	∂_r	$t\partial_r + \partial_u$	$(m - 1)r\partial_r + (m - 1)u\partial_u + 2\rho\partial_\rho + 2mp\partial_p$	∂_p	
0	∂_t	$t\partial_t + r\partial_r$	∂_r	$t\partial_r + \partial_u$	$r\partial_r + u\partial_u - 2\rho\partial_\rho$	$f(p)\partial_p + \rho f'(p)\partial_p$	
$\gamma(p + p_0)$	∂_t	$t\partial_t + r\partial_r$	∂_r	$t\partial_r + \partial_u$	$r\partial_r + u\partial_u - 2\rho\partial_\rho$	$(p + p_0)\partial_p + \rho\partial_p$	
$\frac{n+2}{n}(p + p_0)$	∂_t	$t\partial_t + r\partial_r$	∂_r	$t\partial_r + \partial_u$	$r\partial_r + u\partial_u - 2\rho\partial_\rho$	$(p + p_0)\partial_p + \rho\partial_p$	$t^2\partial_t + tr\partial_r + (r - tu)\partial_u - (n + 2)t(p + p_0)\partial_p - ntp\partial_p$

Table 4: Symmetry classification of bulk moduli (expanded form). Each class of bulk moduli on the left is generated by the symmetry generators given in its row. Here, f is an arbitrary nonzero function, n is 1, 2, or 3 for planar, cylindrical, or spherical symmetry, respectively, a is a nonzero constant, p_0 is an arbitrary constant, and m is an arbitrary constant must be zero in the case $A = A_0\rho^m$. Those columns marked with * apply to planar symmetry only.

A	ϵ	Additional Term	Comment
$(p + p_0)f(\rho)$	$\frac{(p+p_0)g(\rho)}{\rho^2 g'(\rho)} - p_0 \left(1 - \frac{1}{\rho}\right)$	$\frac{p+p_0}{\rho^2 g'(\rho)}$	
$f(\rho)$	$(\rho g(\rho))' - \frac{p}{\rho}$	$p - \rho^2 g'(\rho)$	
$f(p)$	$\frac{1}{\rho} \left(\frac{g(p)}{g'(p)} - p \right)$	$\rho g'(p)$	$g'' \neq 0, f \neq 0$
$A_0 \rho^m$	$\frac{A_0}{m-1} \rho^{m-1} - \frac{p}{\rho}$	$\frac{A_0}{m} \rho^m - p$	$m \neq 0, 1$
$A_0 \rho$	$A_0 - \frac{p}{\rho} + A_0 \log(A_0 \rho)$	$A_0 \rho - p$	
A_0	$-\frac{A_0+p}{\rho}$	$\rho^{A_0} e^{-p}$	
$\gamma(p + p_0)$	$\frac{p+\gamma p_0}{(\gamma-1)\rho}$	$(p + p_0)\rho^{-\gamma}$	$\gamma \neq 1$ constant
$p + p_0$	$\frac{p_0+(p+p_0) \log(p+p_0)}{\rho}$	$\frac{p+p_0}{\rho}$	
$\frac{n+2}{n}(p + p_0)$	$\frac{np+(n+2)p_0}{2\rho}$	$(p + p_0)\rho^{-\frac{n+2}{n}}$	

Table 5: EOS corresponding to various bulk moduli (compact form). The bulk moduli on the left correspond to equations of state where ϵ , the specific internal energy, is of the form given by column 2 plus an arbitrary function of the expression in column 3. For instance, an equation of state corresponding to $A = \gamma p$ is of the form $\frac{p}{(\gamma-1)\rho} + h(p\rho^{-\gamma})$, where h is an arbitrary function. Throughout the table f , and g are arbitrary functions, and n is 1,2, or 3 for planar, cylindrical, or spherical symmetry, respectively, and p_0 and A_0 are constants. The claim is not made that all ϵ corresponding to these A are included in this table, but rather this constitutes an extensive family of examples. For complete information, it is always necessary to convert the EOS into its corresponding bulk modulus on a case-by-case basis.